Series Formulae

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

In general

$$\sum_{i=1}^{n} i^m = \frac{1}{m+1}\left[(n+1)^{m+1} - 1 - \sum_{i=1}^{n}\left((i+1)^{m+1} - i^{m+1} - (m+1)i^m\right)\right]$$

$$\sum_{i=1}^{n-1} i^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k}B_k n^{m+1-k}.$$

Geometric series

$$\sum_{i=0}^{n} c^i = \frac{c^{n+1}-1}{c-1}, c \neq 1, \sum_{i=0}^{\infty} c^i = \frac{1}{1-c}, \sum_{i=1}^{\infty} c^i = \frac{c}{1-c}, |c| < 1,$$

$$\sum_{i=0}^{n} ic^i = \frac{nc^{n+2} - (n+1)c^{n+1} + c}{(c-1)^2}, c \neq 1, \sum_{i=0}^{\infty} ic^i = \frac{c}{(1-c)^2}, |c| < 1.$$

Harmonic series

$$H_n = \sum_{i=1}^{n} \frac{1}{i} \ , \sum_{i=1}^{n} iH_i = \frac{n(n+1)}{2} \ H_n - \frac{n(n-1)}{4} \ .$$

$$\sum_{i=1}^{n} H_i = (n+1)H_n - n, \sum_{i=1}^{n} \left(\frac{i}{m}\right)H_i = \left(\frac{n+1}{m+1}\right)\left(H_{n+1} - \frac{1}{m+1}\right).$$