

Multiplication:

$$C = A \cdot B, \quad c_{i,j} = \sum_{k=1}^n a_{i,k} b_{k,j}$$

Determinants: $\det A \neq 0$ iff A is non-singular.

$$\det A \cdot B = \det A \cdot \det B$$

$$\det A = \sum_{\pi} \prod_{i=1}^n \text{sign}(\pi) a_{i,\pi(i)}$$

2×2 and 3×3 determinant:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = g \begin{vmatrix} b & c \\ e & f \end{vmatrix} - h \begin{vmatrix} a & c \\ d & f \end{vmatrix} + i \begin{vmatrix} a & b \\ d & e \end{vmatrix} \\ = aei + bfg + cdh - ceg - fha - ibd$$

Permanents:

$$\text{perm } A = \sum_{\pi} \prod_{i=1}^n a_{i,\pi(i)}$$